Year 6

NUMBER AND PLACE VALUE

To add, subtract, multiply and divide successfully, pupils need to:
- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- count in multiples of 1 to 10, 25, 50, 100 and 1000, forwards or backwards
- find 10, 100, 1000, 10 000 or 100 000 more or less than a given number
- round any whole number to a required degree of accuracy

DECIMALS

To add, subtract, multiply and divide successfully, pupils need to:
- read, write, order and compare numbers with up to three decimal places
- identify the value of each digit in numbers given to three decimal places
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000, giving the answers up to three decimal places
- recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents
- round decimals with two decimal places to the nearest whole number and to one decimal place

ADDITION

Conceptual understanding and procedural fluency

To add successfully, pupils need to:
- perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations
- practise addition for larger numbers and decimals, using the formal written method of columnar addition
- use knowledge of the order of operations to carry out calculations involving the four operations
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, addition to:
- solve addition multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- solve problems which require answers to be rounded to specified degrees of accuracy

Year 6

ADDITION Continued

Mental strategies

- Continue to use models and images when necessary:
  - decimals addition and subtraction tables

+	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1
0	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1
0·1	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1
0·2	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2
0·3	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3
0·4	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4
0·5	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5
0·6	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6
0·7	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7
0·8	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8
0·9	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8	1·9
1	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2

+	0	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1
0	0	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1
0·01	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11
0·02	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12
0·03	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13
0·04	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14
0·05	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15
0·06	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16
0·07	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17
0·08	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18
0·09	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18	1·19
0·1	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18	1·19	0·2

- Develop further the relationship between addition and subtraction
- Continue to apply knowledge of the commutative law, e.g.
  - put the larger number first and count on in steps of 1, 10, 100 or 1000
  - partition additions into hundreds, tens and ones, then recombine, e.g. 3356 + 257 = 3356 + 200 + 50 + 7

$$= 3556 + 57$$
$$= 3613$$

- Identify near doubles, using doubles already known, e.g. 5·7 + 5·8
- Add the nearest multiple of 10, 100 or 1000, and adjust
- Use patterns of similar calculations, e.g. 9 + 7 = 16 and 0·09 + 0·07 = 0·16
- Use knowledge of the associative law when adding more than two numbers, e.g. 24 + 27 + 16 = (24 + 16) + 27

$$= 40 + 27$$
$$= 67$$

Written methods

Add numbers with more than four digits
- Add decimals with up to three decimal places, including a mix of whole numbers and decimals, and decimals with
- different numbers of decimal places
- Estimate and check the answer to a calculation

Formal written method of columnar addition

456 287 + 359 849

```
  4 5 6 2 8 7
+ 3 5 9 8 4 9
  8 1 6 1 3 6
  1 1 1 1 1
```

57·486 + 45·378

```
  5 7 · 4 8 6
+ 4 5 · 3 7 8
1 0 2 · 8 6 4
    1     1 1
```

Carry digits are recorded below the line, using the words 'carry ten', 'carry one hundred', 'carry one thousand', … not 'carry one'.

Where appropriate, place value columns are labelled, e.g. TO·tht^h, to remind children of the value of each of the digits.

Year 6

SUBTRACTION

Conceptual understanding and procedural fluency

To subtract successfully, pupils need to:
- perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations
- practise subtraction for larger numbers and decimals, using the formal written method of columnar subtraction
- use knowledge of the order of operations to carry out calculations involving the four operations
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

Reason mathematically and solve problems

Pupils need to use and apply their understanding of, and fluency in, subtraction to:
- solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- solve problems which require answers to be rounded to specified degrees of accuracy

Mental strategies

- Continue to use models and images when necessary:
  - decimals addition and subtraction tables

+	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1
0	0	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1
0·1	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1
0·2	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2
0·3	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3
0·4	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4
0·5	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5
0·6	0·6	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6
0·7	0·7	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7
0·8	0·8	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8
0·9	0·9	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8	1·9
1	1	1·1	1·2	1·3	1·4	1·5	1·6	1·7	1·8	1·9	2

+	0	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1
0	0	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1
0·01	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11
0·02	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12
0·03	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13
0·04	0·04	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14
0·05	0·05	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15
0·06	0·06	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16
0·07	0·07	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17
0·08	0·08	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18
0·09	0·09	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18	1·19
0·1	0·1	1·11	1·12	1·13	1·14	1·15	1·16	1·17	1·18	1·19	0·2

- Develop further the relationship between addition and subtraction
- Calculate mentally a difference such as 23 004 – 18 998 by counting up from the smaller to the larger number
- Subtract the nearest multiple of 10, 100 or 1000, and adjust
- Use patterns of similar calculations, e.g. 16 – 9 = 7 and 0·16 – 0·09 = 0·07
- Use partitioning, e.g. 4656 – 358 = 4656 – 300 – 50 – 8

$$= 4356 - 58$$
$$= 4298$$

Year 6

SUBTRACTION Continued

Written methods

- Subtract numbers with more than four digits
- Subtract decimals with up to three decimal places, including a mix of whole numbers and decimals, and decimals with different numbers of decimal places
- Estimate and check the answer to a calculation

Formal written method of columnar subtraction (decomposition)

746 291 – 298 354   63·237 – 45·869

$$
\begin{array}{r}
{}^{6}\!\!\!\not{7}\ {}^{13}\!\!\!\not{4}\ {}^{15}\!\!\!\not{6}\ {}^{12}\!\!\!\not{2}\ {}^{8}\!\!\!\not{9}\ {}^{11}\!\!\!\not{1} \\
-\ 2\ 9\ 8\ \ 3\ 5\ 4 \\
\hline
4\ 4\ 7\ \ 9\ 3\ 7
\end{array}
\qquad
\begin{array}{r}
{}^{5}\!\!\!\not{6}\ {}^{12}\!\!\!\not{3}\cdot{}^{11}\!\!\!\not{2}\ {}^{12}\!\!\!\not{3}\ {}^{17}\!\!\!\not{7} \\
-\ 4\ 5\cdot\ 8\ 6\ 9 \\
\hline
1\ 7\cdot\ 3\ 6\ 8
\end{array}
$$

Start by subtracting the least significant digits first, i.e. in the first example, the ones, then the tens … and finally the hundreds of thousands. Refer to subtracting the tens, for example, by saying '8 tens subtract 5 tens', not '8 subtract 5'.

In the first example, the ones, tens, thousands and tens of thousands to be subtracted are all larger than all of the ones, tens, thousands and tens of thousands you are subtracting from.

The calculation begins by exchanging one of the 9 tens for 10 ones, crossing out the 9 and writing a superscript 8, and crossing out the 1 and writing a superscript 11. The calculation then becomes 11 subtract 4.

You then calculate 8 tens subtract 5 tens.

Next, you exchange one of the 6 thousands for 10 hundreds, crossing out the 6 and writing a superscript 5, and crossing out the 2 and writing a superscript 12. The calculation then becomes 12 hundreds subtract 3 hundreds.

Then you exchange one of the 4 tens of thousands for 10 thousands, crossing out the 4 and writing a superscript 3, and writing a superscript 1 in front of the 5 to make 15 thousands. The calculation then becomes 15 thousands subtract 8 thousands.

Next, you exchange one of the 7 hundreds of thousands for 10 tens of thousands, crossing out the 7 and writing a superscript 6, and writing a superscript 1 in front of the 3 to make 13 tens of thousands. The calculation then becomes 13 tens of thousands subtract 9 tens of thousands.

Then, finally, 600 000 subtract 200 000.

Where appropriate, place value columns are labelled, e.g. TO·tht^h, to remind children of the value of each of the digits.

Year 6

MULTIPLICATION

Conceptual understanding and procedural fluency

To multiply successfully, pupils need to:
- consolidate recall of the multiplication facts for multiplication tables up to 12 × 12
- use known multiplication facts to derive related facts involving multiples of 10, 100 and 1000, and decimals, e.g. 70 × 80 = 5600, 0·8 × 6 = 4·8
- perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations
- continue to multiply whole numbers and those involving decimals by 10, 100 and 1000
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3})
- continue to recognise commutativity in mental calculations
- practise multiplication for larger numbers, using the formal written method of short multiplication
- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- multiply one-digit numbers with up to two decimal places by whole numbers
- multiply numbers with up to two decimal places by one- and two-digit whole numbers
- use knowledge of the order of operations to carry out calculations involving the four operations
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

MULTIPLICATION Continued

Reason mathematically and solve problems

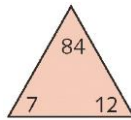Pupils need to use and apply their understanding of, and fluency in, multiplication to:
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- solve problems which require answers to be rounded to specified degrees of accuracy

Mental strategies

- Continue to use models and images when necessary:
  - number lines

1	2	3	4	5	6	7	8	9	10	11	12
12	24	36	48	60	72	84	96	108	120	132	144
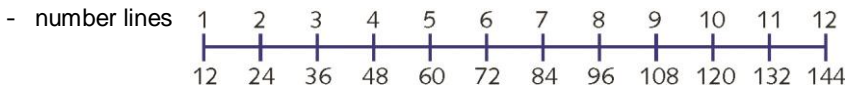
  - trios
    $7 \times 12 = 84$
    $12 \times 7 = 84$
    $84 \div 12 = 7$
    $84 \div 7 = 12$

  - multiplication square to 12 × 12/multiples of 10 multiplication square/decimals multiplication square

×	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

×	20	30	40	50	60	70	80	90	100	110	120
1	20	30	40	50	60	70	80	90	100	110	120
2	40	60	80	100	120	140	160	180	200	220	240
3	60	90	120	150	180	210	240	270	300	330	360
4	80	120	160	200	240	280	320	360	400	440	480
5	100	150	200	250	300	350	400	450	500	550	600
6	120	180	240	300	360	420	480	540	600	660	720
7	140	210	280	350	420	490	560	630	700	770	840
8	160	240	320	400	480	560	640	720	800	880	960
9	180	270	360	450	540	630	720	810	900	990	1080
10	200	300	400	500	600	700	800	900	1000	1100	1200
11	220	330	440	550	660	770	880	990	1100	1210	1320
12	240	360	480	600	720	840	960	1080	1200	1320	1440

×	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2
1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2
2	0·4	0·6	0·8	1	1·2	1·4	1·6	1·8	2	2·2	2·4
3	0·6	0·9	1·2	1·5	1·8	2·1	2·4	2·7	3	3·3	3·6
4	0·8	1·2	1·6	2	2·4	2·8	3·2	3·6	4	4·4	4·8
5	1	1·5	2	2·5	3	3·5	4	4·5	5	5·5	6
6	1·2	1·8	2·4	3	3·6	4·2	4·8	5·4	6	6·6	7·2
7	1·4	2·1	2·8	3·5	4·2	4·9	5·6	6·3	7	7·7	8·4
8	1·6	2·4	3·2	4	4·8	5·6	6·4	7·2	8	8·8	9·6
9	1·8	2·7	3·6	4·5	5·4	6·3	7·2	8·1	9	9·9	10·8
10	2	3	4	5	6	7	8	9	10	11	12
11	2·2	3·3	4·4	5·5	6·6	7·7	8·8	9·9	11	12·1	13·2
12	2·4	3·6	4·8	6	7·2	8·4	9·6	10·8	12	13·2	14·4

Year 6

MULTIPLICATION Continued

- Continue to use the inverse relationship between multiplication and division
- Use related facts and doubling and halving:
  - double or halve the most significant digit first
  - to multiply by 25, multiply by 100 then divide by 4
  - double one number and halve the other
  - find the multiplication facts for the × 24 multiplication table by doubling the × 12 multiplication facts
  - squares of multiples of 10 to 100, e.g. 70 × 70, and the corresponding halves
  - doubles of decimals, e.g. 4·7 × 2, 0·63 × 2, and the corresponding halves
  - doubles of multiples of 10 to 1000, e.g. 830 × 2, and the corresponding halves
  - doubles of multiples of 100 to 10 000, e.g. 48 500 × 2, and the corresponding halves
- Use closely related facts:
  - multiply by 49 or 51 by multiplying by 50 and adjusting
  - develop the × 18 multiplication table by adding facts from the × 10 and × 8 multiplication tables
- Use factors, e.g. 9 × 18 = 9 × 6 × 3
- Use patterns of similar calculations, e.g. 8 × 6 = 48 and 0·8 × 6 = 4·8
- Continue to use and apply the commutative law
- Understand and use the associative law, e.g. 10·6 × 30 = 10·6 × (10 × 3)
  $$\text{or} = (10·6 × 10) × 3$$
- Understand and use the distributive law, e.g. partitioning when multiplying a two-digit or three-digit number by a one-digit number, or two two-digit numbers, and partitioning when multiplying a whole number or decimal by a one-digit number, e.g.

285 × 63 = (200 × 63) + (80 × 63) + (5 × 63)	4·83 × 6 = (4 × 6) + (0·8 × 6) + (0·03 × 6)
= 12 600 + 5040 + 315	= 24 + 4·8 + 0·18
= 17 955	= 28·98

Year 6

MULTIPLICATION Continued

Written methods

- Short multiplication (whole numbers):
  - Multiply multi-digit numbers up to 4 digits by a one-digit whole number
- Estimate and check the answer to a calculation

Formal written method of short multiplication

5643 × 8

```
        5 6 4 3
    ×   5 3 2 8
      4 5 1 4 4
```

Where appropriate, place value columns are labelled, e.g. ThHTO, to remind children of the value of each of the digits.

- Short multiplication (Decimals):
  - Multiply one-digit or two-digit numbers with up to two decimal places by a one-digit number
- Estimate and check the answer to a calculation

Grid method

4·83 × 6

×	4	0·8	0·03	
6	24	4·8	0·18	= 28·98

Expanded written method

4·83 × 6 is equivalent to 483 × 6 ÷ 100

```
          4 · 8 3
    ×           6
            1 8   (   3 × 6)
          4 8 0   (  80 × 6)
        2 4 0 0   (480 × 6)
        2 8 9 8
```

2898 ÷ 100 = 28·98

The first step is to show all of the calculations involved.

Children should describe what they do by referring to the actual values of the digits in the columns, e.g.

- when using the grid method and multiplying the tenths in 4·83 × 6 it is 'zero point eight multiplied by six', not 'eight multiplied by six'
- when using the expanded written method and multiplying the tens in 483 × 6 it is 'eighty multiplied by six, not 'eight multiplied by six'.

Although for both methods the relationship 8 × 6 should be stressed.

Where appropriate, when using the expanded written method, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Formal written method of short multiplication

4·83 × 6 is equivalent to 483 × 6 ÷ 100

```
          4 8 3
    ×     4 1 6
        2 8 9 8
```

2898 ÷ 100 = 28·98

The expanded written method leads to the formal written method of short multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Year 6

MULTIPLICATION Continued

Written methods continued

- Long multiplication (whole numbers):
  - Multiply multi-digit numbers up to four digits by a two-digit number (TO × TO / HTO × TO)
- Estimate and check the answer to a calculation

Grid method

285 × 63

×	200	80	5
60	12 000	4800	300
3	600	240	15

```
        2 8 5
    ×     6 3
17 100 →   1 7 1 0 0
+   855 →       8 5 5
  17 9 5 5 →  1 7 9 5 5
```

The first step is to use the grid method to show all of the calculations involved and how this relates to the expanded written method. For example, 285 is multiplied by 60 (using knowledge of 285 x 6), then 285 is multiplied by 3, and finally the two products are added together.

Expanded written method

Multiplying the <u>most</u> significant digit first

```
      2 8 5
  ×     6 3
  1 7⁵1³0 0   (285 × 60)
      8²5¹5   (285 ×   3)
  1 7 9 5 5
```

Multiplying the <u>least</u> significant digit first

```
      2 8 5
  ×     6 3
      8²5¹5   (285 ×   3)
  1 7⁵1³0 0   (285 × 60)
  1 7 9 5 5
```

The grid method leads to the expanded written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the grid method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Formal written method of long multiplication

285 × 63

```
        2 8 5
    ×     6 3
        8²5¹5
  1 7⁵1³0 0
  1 7 9 5 5
```

The expanded written method leads to the formal written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Year 6

MULTIPLICATION Continued

Written methods continued

- Long multiplication (Decimals):
  - Multiply one-digit numbers with up to two decimal places by a two-digit number
- Estimate and check the answer to a calculation

Method 1: Calculating with decimals

Grid method

$7·56 × 34$

×	7	0·5	0·06
30	210	15	1·8
4	28	2	0·24

$$226·80$$
$$+ \quad 30·24$$
$$257·04$$
$$\scriptstyle 1$$

The first step is to show all of the calculations involved, e.g. 7·56 is multiplied by 30 (using knowledge of 756 x 3), then 7·56 is multiplied by 4, and finally the two products are added together.

Method 2: Converting decimals to whole numbers before calculating, then converting the answer back to decimals

Expanded written method

Multiplying the <u>most</u> significant digit first

$7·56 × 34$ is equivalent to $756 × 34 ÷ 100$

```
      7 5 6
  ×     3 4
  2 2¹6¹8 0   (756 × 30)
    3 0²2²4   (756 ×  4)
  2 5 7 0 4
        1
```

$25\ 704 ÷ 100 = 257·04$

Multiplying the <u>least</u> significant digit first

$7·56 × 34$ is equivalent to $756 × 34 ÷ 100$

```
      7 5 6
  ×     3 4
    3 0²2²4   (756 ×  4)
  2 2¹6¹8 0   (756 × 30)
  2 5 7 0 4
        1
```

$25\ 704 ÷ 100 = 257·04$

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Formal written method of long multiplication

$7·56 × 34$ is equivalent to $756 × 34 ÷ 100$

```
      7 5 6
  ×     3 4
    3 0²2²4
  2 2¹6¹8 0
  2 5 7 0 4
        1
```

$25\ 704 ÷ 100 = 257·04$

The expanded written method leads to the formal written method of long multiplication so that children fully understand the procedure, and the effectiveness and efficiency of the method.

The amount of time that should be spent teaching and practising the expanded written method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

Where appropriate, place value columns are labelled, e.g. HTO, to remind children of the value of each of the digits.

Year 6

DIVISION

Conceptual understanding and procedural fluency

To divide successfully, pupils need to:
- consolidate recall of the division facts for multiplication tables up to 12 × 12
- use known division facts to derive related facts involving multiples of 10, 100 and 1000, and decimals, e.g. 6300 ÷ 90 = 70, 6·3 ÷ 9 = 0·7
- perform mental calculations, including with mixed operations, large numbers, decimals and more complex calculations
- continue to divide whole numbers and those involving decimals by 10, 100 and 1000, giving the answers up to three decimal places
- identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers
- know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- practise division for larger number, using the formal written method of short division
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- divide numbers with up to two decimal places by one- and two-digit whole numbers
- use knowledge of the order of operations to carry out calculations involving the four operations
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy

Reason mathematically and solve problems

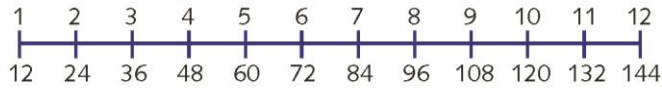Pupils need to use and apply their understanding of, and fluency in, division to:
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- solve problems which require answers to be rounded to specified degrees of accuracy

Year 6

DIVISION Continued

- Continue to use models and images when necessary:
  - number lines

$$\begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 12 & 24 & 36 & 48 & 60 & 72 & 84 & 96 & 108 & 120 & 132 & 144 \end{array}$$

  - multiplication square to 12 × 12/multiples of 10 multiplication square/decimals multiplication square

×	2	3	4	5	6	7	8	9	10	11	12
1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

×	20	30	40	50	60	70	80	90	100	110	120
1	20	30	40	50	60	70	80	90	100	110	120
2	40	60	80	100	120	140	160	180	200	220	240
3	60	90	120	150	180	210	240	270	300	330	360
4	80	120	160	200	240	280	320	360	400	440	480
5	100	150	200	250	300	350	400	450	500	550	600
6	120	180	240	300	360	420	480	540	600	660	720
7	140	210	280	350	420	490	560	630	700	770	840
8	160	240	320	400	480	560	640	720	800	880	960
9	180	270	360	450	540	630	720	810	900	990	1080
10	200	300	400	500	600	700	800	900	1000	1100	1200
11	220	330	440	550	660	770	880	990	1100	1210	1320
12	240	360	480	600	720	840	960	1080	1200	1320	1440

×	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2
1	0·2	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1	1·1	1·2
2	0·4	0·6	0·8	1	1·2	1·4	1·6	1·8	2	2·2	2·4
3	0·6	0·9	1·2	1·5	1·8	2·1	2·4	2·7	3	3·3	3·6
4	0·8	1·2	1·6	2	2·4	2·8	3·2	3·6	4	4·4	4·8
5	1	1·5	2	2·5	3	3·5	4	4·5	5	5·5	6
6	1·2	1·8	2·4	3	3·6	4·2	4·8	5·4	6	6·6	7·2
7	1·4	2·1	2·8	3·5	4·2	4·9	5·6	6·3	7	7·7	8·4
8	1·6	2·4	3·2	4	4·8	5·6	6·4	7·2	8	8·8	9·6
9	1·8	2·7	3·6	4·5	5·4	6·3	7·2	8·1	9	9·9	10·9
10	2	3	4	5	6	7	8	9	10	11	12
11	2·2	3·3	4·4	5·5	6·6	7·7	8·8	9·9	11	12·1	13·2
12	2·4	3·6	4·8	6	7·2	8·4	9·6	10·8	12	13·2	14·4

- Continue to use the inverse relationship between multiplication and division
- Continue to use halving, e.g. connect the 3, 6 and 12 multiplication tables
- Understand and use the distributive law, e.g. partitioning when dividing a three-digit number by a one-digit number, e.g.

$$486 \div 9 = (450 \div 9) + (36 \div 9)$$
$$= 50 + 4$$
$$= 54$$

DIVISION Continued

Written methods

- Short division, including with remainders expressed as a whole number, fraction or decimal (whole numbers)
  - Divide numbers up to 4 digits by a one-digit number (HTO ÷ O/ThHTO ÷ O)
- Estimate and check the answer to a calculation

Formal written method of short division

Whole number remainder

$1838 \div 8$

$$
\begin{array}{r}
2\ 2\ \ 9\ r\ 6 \\
\hline
8\overline{)1\ 8\ ^23\ ^78}
\end{array}
$$

Fraction remainder

$1838 \div 8$

$$
\begin{array}{r}
2\ \ 2\ \ 9\ \tfrac{3}{4} \\
\hline
8\overline{)1\ 8\ ^23\ ^78}
\end{array}
$$

Decimal remainder

$1838 \div 8$

$$
\begin{array}{r}
2\ \ 2\ \ 9\ .\ 7\ 5 \\
\hline
8\overline{)1\ 8\ ^23\ ^78\ .^60\ ^40}
\end{array}
$$

Children should describe what they are doing using phrases similar to the following: 'How many eights divide into 1800 so that the answer is a multiple of 100?' (200) There are 200 eights or 1600, with 200 remaining. The superscript 2 represents the 2 hundreds that are remaining after 8 has been divided into 1800. It is written in front of the 3 to show that a total of 23 tens (230) now have to be divided by eight.

Children then ask: 'How many eights divide into 230 so that the answer is a multiple of 10?' (20) There are 20 eights or 160, with 70 remaining. The superscript 7 represents the 7 tens that are remaining after 8 has been divided into 230. It is written in front

of the 8 to show that 78 now has to be divided by 8.

Children then ask: 'How many eights in 78?' (9 remainder 6). Depending on the context, the remainder is written as a whole number, fraction, decimal or rounded up or down.

- Short division (Decimals)
  - Divide numbers with up to two decimal places by a one-digit number (O·th ÷ O/TO·th ÷ O)
- Estimate and check the answer to a calculation

Method 1: Calculating with decimals

$$
\begin{array}{r}
7\ .\ 5\ 6 \\
\hline
6\overline{)4\ 5\ .^33\ ^36}
\end{array}
$$

Method 2: Converting decimals to whole numbers before calculating, then converting the answer back to decimals

45·36 ÷ 6 is equivalent to 4536 ÷ 6 ÷ 100

$$
\begin{array}{r}
7\ \ 5\ 6 \\
\hline
6\overline{)4\ 5\ ^33\ ^36}
\end{array}
$$

756 ÷ 100 = 7·56

Phrases similar to those above for short division of whole numbers should be used for short division involving decimals.

An emphasis should be placed on recognising the value of each of the digits in the dividend.

- Long division, including with remainders expressed as a whole number, fraction or decimal (Whole numbers)
  - Divide numbers up to 4 digits by a two-digit number (HTO ÷ TO/ThHTO ÷ TO)
- Estimate and check the answer to a calculation

Expanded written method of long division

$$
\begin{array}{r}
3\ 2\ 4\ r\ 4 \\
\hline
18\overline{)5\ 8\ 3\ 6} \\
-\ 5\ 4\ 0\ 0 \quad (300 \times 18) \\
\hline
^{34}1^{13}3\ 6 \\
-\ \ 3\ 6\ 0 \quad (20 \times 18) \\
\hline
0\ 7\ 6 \\
-\ 0\ 7\ 2 \quad (4 \times 18) \\
\hline
4
\end{array}
$$

5836 ÷ 18 = 324 r 4 or $324\tfrac{2}{9}$

Formal written method of long division

$$
\begin{array}{r}
3\ 2\ 4\ r\ 4 \\
\hline
18\overline{)5\ 8\ 3\ 6} \\
-\ \underline{5\ 4}\downarrow \\
4\ 3 \\
-\ \underline{3\ 6}\downarrow \\
7\ 6 \\
-\ \underline{7\ 2} \\
4
\end{array}
$$

5836 ÷ 18 = 324 r 4 or $324\tfrac{2}{9}$

The amount of time that should be spent teaching and practising the expanded written method of long division will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10 and 100, with subtracting multiples of 10 and 100 mentally, and in their understanding of place value.

DIVISION Continued

Written methods continued

- Long division (Decimals)
  - Divide numbers with up to two decimal places by a two-digit whole number (TO·th ÷ TO)
- Estimate and check the answer to a calculation

Method 1: Calculating with decimals

Expanded written method of long division

58·32 ÷ 18

```
          3 □ 2 4
    18) 5 8 □ 3 2
      − 5 4 □ 0 0    (   3 × 18)
        ³4 □⁴³3 2
      −   3 □ 6 0    ( 0.2 × 18)
          0 □ 7 2
      −   0 □ 7 2    (0.04 × 18)
          0 □ 0 0
```

Formal written method of long division

58·32 ÷ 18

```
          3 □ 2 4
    18) 5 8 □ 3 2
      − 5 4 □↓
          4 □ 3
      −   3 □ 6↓
          0 □ 7 2
      −   0 □ 7 2
                0
```

The amount of time that should be spent teaching and practising this expanded written method of long division (i.e Method 1) will depend on how secure the children are in their recall of multiplication and division facts, including involving decimals with up to two decimal places, with subtracting whole and decimal numbers mentally, and in their understanding of place value.

Method 2: Converting decimals to whole numbers before calculating, then converting the answer back to decimals

Expanded written method of long division

58·32 ÷ 18 is equivalent to 5832 ÷ 18 ÷ 100

```
          3 2 4
    18) 5 8 3 2
      − 5 4 0 0    (300 × 18)
        ³4⁴3 2
      −   3 6 0    ( 20 × 18)
          0 7 2
      −   0 7 2    (  4 × 18)
              0
```

324 ÷ 100 = 3.24

Formal written method of long division

58·32 ÷ 18 is equivalent to 5832 ÷ 18 ÷ 100

```
          3 2 4
    18) 5 8 3 2
      − 5 4↓
          4 3
      −   3 6↓
            7 2
      −     7 2
              0
```

324 ÷ 100 = 3.24

The amount of time that should be spent teaching and practising this expanded written method of long division (i.e. Method 2) will depend on how secure the children are in their recall of multiplication and division facts, including involving multiples of 10 and 100, with subtracting multiples of 10 and 100 mentally, and in their understanding of place value.